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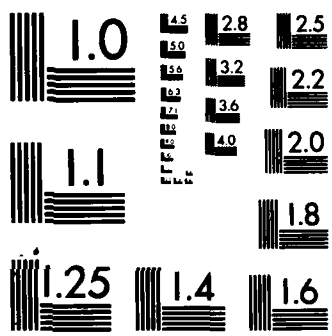
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THE BEHAVIOR OF STREAMLINES NEAR SEPARATION POINT AND SEPARATION CRITERION

by

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FOREIGN TECHNOLOGY DIVISION

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THE BEHAVIOR OF STREAMLINES NEAR SEPARATION POINT AND SEPARATION CRITERION

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Abstract

This paper studies the behavior of streamlines near separation point and on this basis discusses the separation criterion of two-dimensional non-compressible steady and unsteady flow. It is pointed out that the MRS criterion is necessary when the separation flow has two lines with u=0 passing through the separation point. For the flow described by boundary layer equations, the separation point is generally a Goldstein singularity and the MRS criterion is generally equivalent to the singularity criterion. However, the separation point is not a singularity for the flow described by Navier-Stokes equations.

Introduction

Recently, academic circles have been paying very close attention to the separation criteria of two-dimensional unsteady viscous flows. Early during the 1950's, Moore [1], Rott [2] and Sears [3] separately researched this problem and after independently drawing it up people used the "MRS" criterion formed from their names. This criterion considers that in a coordinate system which moves together with the separation point, the conditions of the flow having separation are: $u_0 - \left(\frac{\partial u}{\partial y}\right)_0 \rightarrow 0$. Here, u is the speed of the gas moving along the material surface, y is the coordinate vertical to the material surface and subscript "0" indicates the value of the separation point area. Afterwards, based on their calculations, Sears and Telionis [4] discovered that the separation point is a Goldstein singularity in the unsteady boundary layer and therefore they

popularized this concept and proposed using non-singularity to determine the criterion of whether or not the flow is separated. In recent work, O'Brien [5] studied zero vortex criterion and considered that the considition occuring at separation is:

$$u_0 = 0,\ \Omega_0 = \left(\frac{\partial v}{\partial x}\right)_0 - \left(\frac{\partial u}{\partial y}\right)_0 = 0.$$

Here, x is the coordinate along the material surface and v is the speed component of the y direction. K.C. Wang [6] studied the problem of separation criteria yet as pointed out in reference [7], the above mentioned separation criterion is not universally applicable. Recently, reference [8] studied the problem of separation criteria and established a criterion with universal application. Based on the research of reference [8], the two-dimensional viscous separation flow can basically be divided into two types based on the flow pattern near the separation point: the first type has one line with u=0 passing through the separation point and the second type has two lines with u=0 passing through the separation point. The former is the flow which occurs in the adverse current movement wall and the latter is the flow which appears when there is a fair current movement wall and steady state (Figs. 1 and 2). Reference [8] points out that $\left(\frac{\partial u}{\partial y}\right)_0=0$ is not a necessary condition of separation for

the first type of flow. This paper is a supplement to reference [8] and it shows that for the second type of separation flow, $\left(\frac{\partial u}{\partial y}\right)_s = 0$ is a necessary condition for separation and in the approximate range of the boundary layer the separation points are generally singular points and the MRS criterion is correct. However, the separation is not singular for the flow described by the Navier-Stokes equations.

For purposes of simplification, this paper discusses the non-compressible two-dimensional viscous flow.

II. Separation of Steady Viscous Flow

Because the separation point is the point of intersection of

the streamline, it can only occur in a position where the speed is zero. Therefore, the separation point for the steady viscous flow stagnating on the material surface is located on the material surface. Figure 1 is a drawing of its separation. We studied the behavior of the limit streamline in the separation point area when $y \rightarrow 0$.

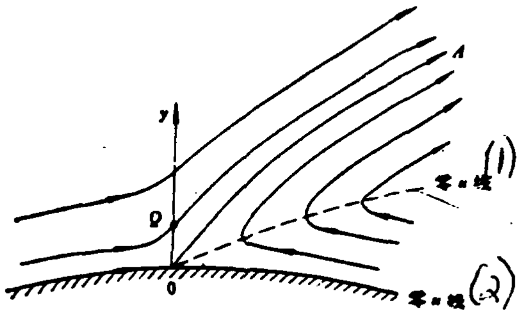

Fig. 1 Drawing of separation of steady non-compressible boundary layer.

Key: (1)-(2) u=0 line.

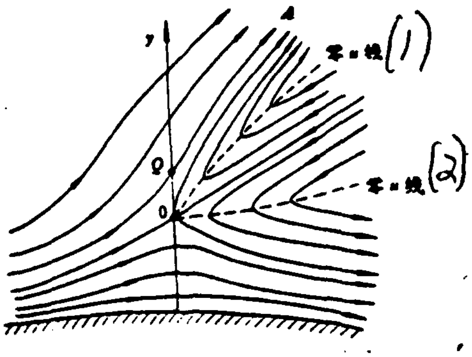

Fig. 2 Drawing of separation of unsteady non-compressible boundary layer.

Key: (1)-(2) u=0 line.

1. Steady Boundary Line

As shown in Fig. 1, because the two lines with u=0 and different slopes pass through the 0 point, along its two lines we separately differentiate the u and attain:

$$\left.\begin{aligned}\left(\frac{\partial u}{\partial x}\right)_0 + \left(\frac{dy}{dx}\right)_{u1}\left(\frac{\partial u}{\partial y}\right)_0 = 0\\ \left(\frac{\partial u}{\partial x}\right)_0 + \left(\frac{dy}{dx}\right)_{u2}\left(\frac{\partial u}{\partial y}\right)_0 = 0\end{aligned}\right\} \tag{2.1}$$

Here, $\left(\frac{dy}{dx}\right)_{u1}$ and $\left(\frac{dy}{dx}\right)_{u2}$ are separately the slopes of the two lines with u=0 at the 0 point. Because $\left(\frac{dy}{dx}\right)_{u1} \neq \left(\frac{dy}{dx}\right)_{u2}$, formula (2.1) is given as:

$$\left(\frac{\partial u}{\partial x}\right)_0 = \left(\frac{\partial u}{\partial y}\right)_0 = 0 \tag{2.2}$$

Therefore, $\left(\frac{\partial u}{\partial y}\right)_0=0$ is the necessary condition of separation. The equation of the streamline near the o point is:

$$\frac{dy}{dx} = \frac{v}{u} \tag{2.3}$$

At the separation point $u_0=v_0=0$ and therefore the slope of the OA streamline (see Fig. 1) at the 0 point is a zero-to-zero form. We now pass through the 0 point to make the y axis and on it we select a Q point at a distance from the 0 point of Δy. We study the situation when $\Delta y \rightarrow 0$ (Q point $\rightarrow$ 0 point) and the slope of the limit streamline. Based on the L'Hopital law, it can be indicated as:

$$\left(\frac{dy}{dx}\right)_0 = \frac{\left(\frac{\partial v}{\partial y}\right)_0}{\left(\frac{\partial u}{\partial y}\right)_0} \tag{2.4}$$

Based on the continuous equation among the group of boundary

equations, $\left(\frac{\partial v}{\partial y}\right)_0 = -\left(\frac{\partial u}{\partial x}\right)_0$. We substitute this formula into (2.4) and we can know from (2.2) that (2.4) is also a zero-to-zero form. By using the L'Hopital law we can then obtain the slope of the limit streamline:

$$\left(\frac{dy}{dx}\right)_0 = \frac{\left(\frac{\partial^2 v}{\partial y^2}\right)_0}{\left(\frac{\partial^2 u}{\partial y^2}\right)_0} = -\frac{\left[\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right)\right]_0}{\left(\frac{\partial^2 u}{\partial y^2}\right)_0} \qquad (2.5)$$

Here, we once again use the continuous equation and based on the momentum equation of the boundary layer it is easy to derive:

$$\left(\frac{\partial^2 u}{\partial y^2}\right)_0 = \frac{1}{\mu}\left(\frac{\partial p}{\partial x}\right)_0$$

However, based on the definition of local frictional resistance $\tau_w = \mu \frac{\partial u}{\partial y}$, $\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right) = \frac{1}{\mu}\frac{\partial \tau_w}{\partial x}$, and substituting in (2.5) we obtain:

$$\left(\frac{dy}{dx}\right)_0 = -\frac{\left(\frac{d\tau_w}{dx}\right)_0}{\left(\frac{dp}{dx}\right)_0} \qquad (2.6)$$

Because $\left(\frac{dp}{dx}\right)_0$ is generally a non-zero limited value in the separation point area, this formula shows that if $\left(\frac{d\tau_w}{dx}\right)_0 = \infty$, that is, the 0 point is a Goldstein singularity, then $\left(\frac{dy}{dx}\right)_0 \longrightarrow \infty$ and the limit streamline is vertical to the material surface at the 0 point. If $\left(\frac{d\tau_w}{dx}\right)_0$ is a limited value, that is the 0 point is not a singularity, at this time, the limit streamline separates from the material surface along the inclined direction. This shows that studying the singularity of the 0 point is equivalent to studying the angle of inclination of this point's limit streamline.

On the other hand, the momentum equation of the boundary layer continuously carries out differentiation two times on the y and after using the continuous equation we obtain:

$$\left(\frac{\partial u}{\partial y}\right)\frac{\partial^2 u}{\partial x \partial y} + u\,\frac{\partial^3 u}{\partial x \partial y^2} + v\,\frac{\partial^3 v}{\partial y^3} - \frac{\partial u}{\partial x}\,\frac{\partial^2 u}{\partial y^2} = \nu\,\frac{\partial^4 u}{\partial y^4}$$

Because u=v=0 on the material surface and $\frac{\partial}{\partial x}\left(\frac{\partial^2 u}{\partial y^2}\right)$, $\frac{\partial^2 u}{\partial y^2}$, $\frac{\partial^3 u}{\partial y^3}$, is limited, therefore:

$$\frac{\partial u}{\partial y}\,\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right) = \nu\,\frac{\partial^4 u}{\partial y^4} \tag{2.7}$$

We can see that if $\left(\frac{\partial^4 u}{\partial y^4}\right)_o$ is not zero at the 0 point, because $\left(\frac{\partial u}{\partial y}\right)_0 = 0$, $\left[\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right)\right]_0 \to \infty$, that is the separation point is a singular point, if $\left(\frac{\partial^4 u}{dy^4}\right)_o$ is zero, then $\left[\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right)_o\right]$ is a limited value, that is the separation point is non-singular. Aside from special situations (e.g. the similar flow of outflow $u_c = Cx^m$, m=0.0904), the $\left(\frac{\partial^4 u}{\partial y^4}\right)_o$ of most boundary layers are not zero and therefore, generally speaking, the separation of steady boundary layers is singular.

2. Flow Described by Navier-Stokes Equations

Given this type of situation, formulas (2.5) and (2.6) are still tenable. However, we know from the Navier-Stokes equation that

$$\left.\begin{aligned} \left(\frac{\partial^2 u}{\partial y^2}\right)_0 &= \frac{1}{\mu}\left(\frac{\partial p}{\partial x}\right)_0 \\ \left(\frac{\partial^2 v}{\partial y^2}\right)_0 &= -\left[\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right)\right]_0 = -\frac{1}{\mu}\left(\frac{\partial \tau_w}{\partial x}\right)_0 = \frac{1}{\mu}\left(\frac{\partial p}{\partial y}\right)_0 \end{aligned}\right\} \tag{2.8}$$

Therefore, by substituting in formula (2.5) or (2.6) we obtain

$$\left(\frac{dx}{dy}\right)_0 = -\frac{\left(\frac{\partial p}{\partial y}\right)_0}{\left(\frac{\partial p}{\partial x}\right)_0} \tag{2.9}$$

Therefore, in most situations $\left(\frac{\partial p}{\partial x}\right)_o$ and $\left(\frac{\partial p}{\partial y}\right)_o$ are limited and because of this the slope of the limit streamline at the 0 point is limited and the separation point is non-singular. This is very different from the situation of the boundary layer.

III. Separation of Unsteady Viscous Flow

We first study the flow described by the non-compressible boundary layer equations. The group of basic equations is:

$$\left.\begin{aligned} &\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \\ &\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} \end{aligned}\right\} \quad (3.1)$$

We will now discuss the wall surface movement.

Because the streamline is composed of instantaneous flow field speed distribution, the form of the streamline is interrelated with the reference coordinate system of the described movement. If the streamline appears to intersect at a certain point, then the speed of this point is necessarily xero for the reference coordinate system. Therefore, only when the reference coordinate system moves with the separation point can we see the separation picture as shown in Fig. 2. Assuming it is in the x,y coordinate system, the speed of separation point 0's movement is u_s, v_s and we introduce:

$$\left.\begin{aligned} x &= \xi + \int_0^\tau u_s(t)\,dt \\ y &= \eta + \int_0^\tau v_s(t)\,dt \\ t &= \tau \end{aligned}\right\} \quad (3.2)$$

Naturally

$$\left.\begin{aligned}\frac{\partial}{\partial\xi}&=\frac{\partial}{\partial x}\\ \frac{\partial}{\partial\eta}&=\frac{\partial}{\partial y}\\ \frac{\partial}{\partial\tau}&=\frac{\partial}{\partial t}+u_s\frac{\partial}{\partial x}+v_s\frac{\partial}{\partial y}\end{aligned}\right\}$$

By substituting (3.2) into (3.1) we obtain

$$\left.\begin{aligned}&\frac{\partial\bar{u}}{\partial\xi}+\frac{\partial\bar{v}}{\partial\eta}=0\\ &\frac{\partial\bar{u}}{\partial\tau}+\bar{u}\frac{\partial\bar{u}}{\partial\xi}+\bar{v}\frac{\partial\bar{u}}{\partial\eta}=-F+\nu\frac{\partial^2\bar{u}}{\partial\eta^2}\end{aligned}\right\} \tag{3.3}$$

In the formula

$$\left.\begin{aligned}\bar{u}&=u-u_s(\tau)\\ \bar{v}&=v-v_s(\tau)\\ F&=\frac{1}{\rho}\frac{\partial p}{\partial\xi}+\frac{du_s}{d\tau}\end{aligned}\right\} \tag{3.4}$$

As shown in Fig. 2, in the ξ, η coordinate system, point 0 is non-moving, that is $\bar{u}_o=\bar{v}_o=0$. Based on the hypothesis, when its rear reflux area has two lines with $\bar{u}=0$ and there is differentiation of $\bar{u}$ along the two lines, by using a method similar to the one in the above section, we can obtain:

$$\frac{\partial\bar{u}}{\partial\xi}=\frac{\partial\bar{u}}{\partial\eta}=0 \tag{3.5}$$

This shows that $\frac{\partial\bar{u}}{\partial\eta}=\frac{\partial u}{\partial y}$ is also a necessary condition of the unsteady second type of separation. In the ξ, η coordinate system, the equation of the streamline near the 0 point is:

$$\frac{d\eta}{d\xi}=\frac{\bar{v}}{\bar{u}} \tag{3.6}$$

Because $\bar{u}_o=\bar{v}_o=0$, this formula presents the zero-to-zero form at the 0 point. By using the L'Hopital law to find the slope of the limit streamline of $\eta\to 0$, we can obtain

$$\left(\frac{d\eta}{d\xi}\right)_0 = \frac{\left(\frac{\partial \bar{v}}{\partial \eta}\right)_0}{\left(\frac{\partial \bar{u}}{\partial \eta}\right)_0} = -\frac{\left(\frac{\partial \bar{u}}{\partial \xi}\right)_0}{\left(\frac{\partial \bar{u}}{\partial \eta}\right)_0} \tag{3.7}$$

Here, we used the first formula of (3.3). We can know from (3.5) that (3.7) is still a zero-to-zero form, and then after using the L'Hopital law we obtain:

$$\left(\frac{d\eta}{d\xi}\right)_0 = -\frac{\left[\frac{\partial}{\partial \xi}\left(\frac{\partial \bar{u}}{\partial \eta}\right)\right]_0}{\left(\frac{\partial^2 \bar{u}}{\partial \eta^2}\right)_0} \tag{3.8}$$

From the second formula of (3.3) we obtain:

$$\frac{\partial^2 \bar{u}}{\partial \eta^2} = \frac{\rho F}{\mu}$$

On the other hand, if formula 2 of (3.3) continuously differentiates η two times and we use formula 1 of (3.3), we obtain:

$$\frac{\partial}{\partial \tau}\left(\frac{\partial^2 \bar{u}}{\partial \eta^2}\right) + \frac{\partial \bar{u}}{\partial \eta}\frac{\partial^2 \bar{u}}{\partial \xi \partial \eta} + \bar{u}\frac{\partial}{\partial \xi}\left(\frac{\partial^2 \bar{u}}{\partial \eta^2}\right) + \bar{v}\frac{\partial^3 \bar{u}}{\partial \eta^3} - \frac{\partial \bar{u}}{\partial \xi}\frac{\partial^2 \bar{u}}{\partial \eta^2} = \nu \frac{\partial^4 \bar{u}}{\partial \eta^4}$$

We use conditions $\bar{u}_o=\bar{v}_o=0$ and (3.5), and because $\frac{\partial}{\partial \xi}\left(\frac{\partial^2 \bar{u}}{\partial \eta^2}\right)$, $\frac{\partial^3 \bar{u}}{\partial \eta^3}$, $\frac{\partial^2 \bar{u}}{\partial \eta^2}$ are both limited values, the above formula is given as:

$$\left(\frac{\partial \bar{u}}{\partial \eta}\right)_0\left(\frac{\partial^2 \bar{u}}{\partial \xi \partial \eta}\right)_0 = G_0 \tag{3.9}$$

In the formula

$$G_0 = \nu\left(\frac{\partial^4 \bar{u}}{\partial \eta^4}\right)_0 - \frac{d}{d\tau}\left(\frac{\rho F}{\mu}\right) \tag{3.10}$$

(3.9) shows that if $G_o \neq 0$, then $\left[\frac{\partial}{\partial \xi}\left(\frac{\partial \bar{u}}{\partial \eta}\right)\right]_0 \to \infty$
that is the 0 point is the singularity of the flow. Moreover, we can know from (3.8) that at this time the slope of the limit streamline at the 0 point is infinitely great. If $G_o=0$, then $\left[\frac{\partial}{\partial \xi}\left(\frac{\partial \bar{u}}{\partial \eta}\right)\right]_o$ is a limited value. At this time, the 0 point is not the singular point of the flow and the slope of the limit streamline is limited. Aside from special situations, G_o is generally not zero and therefore the second type of separation point of the unsteady boundary layer flow is generally singular.

As regards the flow described by the Navier-Stokes equations, by using a similar method, we can prove that the separation point is non-singular.

IV. Discussion and Conclusion

Based on the above analysis, when there is a separation flow with two lines having u=0 passing the separation point, we can carry out the following discussion and make the following conclusions:

1. If the flow is separated at the 0 point, it is necessary to have $u_o=\left(\frac{\partial u}{\partial y}\right)_o=0$ and $v_o=\left(\frac{\partial u}{\partial x}\right)_o=0$ and therefore the conditions brought forth by Moore, Rott and Sears are necessary.

2. If the flow is satisfied at the 0 point:

$$u_0 = v_0 = \left(\frac{\partial u}{\partial x}\right)_0 = \left(\frac{\partial u}{\partial y}\right)_0 = 0$$

In the flow described by the boundary layer equations, the limit streamline has an infinitely large slope at the 0 point. That is, the flow has sudden deflection at the 0 point. This signifies the appearance of separation. Generally speaking, at this time, the separation point is a Goldstein singularity and the MRS criterion is equivalent to the singularity criterion.

3. As regards the flow described by the NS equations, the separation point is not a singularity and the singularity separation criterion is not applicable.

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